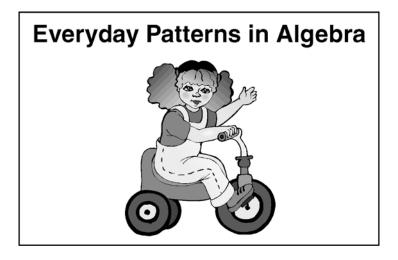
SITES-M Mathematics Challenge



Level: Grade Four

Standard: Algebra

Learning Target: Focus on Patterns

Checks for Understanding

0406.3.3 Create, explain and use a rule to generate terms of a pattern or sequence.

State Performance Indicators

SPI 0406.3.1	Use letters and symbols to represent an unknown quantity
	and write a simple mathematical expression.
SPI 0406.3.2	Make generalizations about geometric and numeric patterns.
SPI 0406.3.3	Represent and analyze patterns using words and function
	tables.

SITES-M Mathematics Challenge Grade 4–Focus on Patterns Everyday Patterns in Algebra

The purpose of the Mathematics Challenges is to provide opportunities for students to develop and demonstrate understanding of important mathematical concepts and standards. Each Challenge includes a set of tasks that require higher-order thinking skills. Because these types of tasks may be new for students and they will have varying levels of understanding, the student responses will vary. The Challenges and guiding questions were designed to help teachers plan their implementation and elicit, analyze, and act on evidence of student understanding.

You will be able to choose which Mathematics Challenge Packet to implement each month, according to the learning needs of your students and your teaching context. Each packet contains all the materials necessary to implement the Mathematics Challenge including a grade-appropriate Challenge, the Mathematics Challenge Meeting Protocol, and the Guiding Questions for Analyzing Student Responses to Mathematics Challenges.

For each Challenge, you will complete a six step process of planning, implementation, and analysis and reflection.

Stage	Step	Task	
	Step 1.	Review the Mathematics Challenge Meeting Protocol	
Planning	Step 2.	Review and solve the Mathematics Challenge prior to your Professional Learning Community (PLC) meeting. Think about your responses to the guiding questions on the Meeting Protocol	
	Step 3.	Hold your PLC meeting and discuss your responses to the Guiding Questions on the Meeting Protocol	
Implementation	Step 4.	Implement the Mathematics Challenge with your class	
	Step 5.	For your own planning and documentation, respond to the Guiding Questions on the Analyzing Student Responses Protocol	
Analysis and Reflection	Step 6.	To help us improve the Challenges and to provide recommendations for teachers implementing them in future years, complete the Mathematics Challenge Feedback Log and provide copies of all student work to the Assessment Coordinator	

The Mathematics Challenge Process

SITES-M Mathematics Challenge Grade 4–Focus on Patterns Everyday Patterns in Algebra

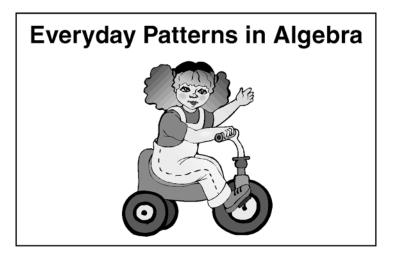
Mathematics Challenge Meeting Protocol

Each month, your Professional Learning Community will meet to discuss the implementation of one Mathematics Challenge. In preparation for your monthly meeting, please print and review this month's Mathematics Challenge, solve all tasks within the Challenge, and think about the guiding questions below. These questions will be used to facilitate a group discussion regarding the implementation of the upcoming Mathematics Challenge.

Guiding Questions for Implementing the Mathematics Challenges

- 1. What is the title of the Challenge that you will use this month?
- 2. What skills or standards is this Challenge measuring?
- 3. Where does this Challenge fit within your curriculum? Within which unit?
- 4. At what point during the unit will you administer this Challenge (e.g., At the beginning of a unit to determine what students do or do not know, at the end of a unit to assess what students have or have not learned, in the middle of a unit to determine where to go next instructionally)?
- 5. How will your students complete this Challenge (e.g., individually, one-on-one, in small groups, as a class)? Why?
- 6. Are there any prerequisite skills, common misunderstandings, or vocabulary needs that you will have to address? What are they?
- 7. What difficulties do you anticipate your students will have with the Challenge? How will you address them?
- 8. Are these skills and difficulties different for special needs students, ELL students, etc.? How? Will you do anything different for these students? What?
- 9. How will you evaluate student responses (e.g., grade responses with the provided rubric, scan responses to identify common mistakes/misconceptions, have students evaluate one another's responses, have students evaluate their own response)?
- 10. What will student responses to this Challenge tell you about student understanding?
- 11. How might you use this evidence of student understanding to adapt your teaching and learning?
- 12. What other materials, resources, or support might you need? Where can you get them?
- 13. How can your colleagues assist you in the analysis of student understanding?
- 14. What other questions or concerns do you have about this Mathematics Challenge?

After you have implemented the challenge with your class, be sure to respond to the Guiding Questions on the Analyzing Student Responses Protocol.



Standard: Algebra

Learning Target: Focus on Patterns

Claims:

Students should understand and be able to explain or demonstrate how to:

- Make generalizations about geometric and numeric patterns;
- ✓ Create, explain and use a rule to generate terms of a pattern or sequence;
- ✓ Represent and analyze patterns using words and function tables;
- Use letters and symbols to represent an unknown quantity and write a simple mathematical expression.

Task Preparation:

Each student will need a copy of the Student Response Sheet and a pencil.

Stimulus Cards (Drawing or Word Description):

None

Manipulatives/Supplies:

Pencils

Cues/Directions:

Distribute student response sheets. Students should be directed to look carefully at each figure. Allow students time to answer.

Instruct students to follow along as you read aloud and say: **Ms. Randolph's** class is helping the local preschool build a path for children to ride their tricycles. The path has two types of square shapes that make a pattern. Have students look at the two shapes. Part of the path is shown below. Have students look at the part of the path shown.

- Say: Circle the set of square shapes that should go in the blanks to continue the pattern. (TEACHER NOTE: Students should circle the correct set of shapes.) Describe the pattern of the path. (TEACHER NOTE: Students should write their descriptions in the box.)
- 2. Another part of the path is shown below. (TEACHER NOTE: Have students look at the part of the path shown.) Can the part below be used as a pattern in the path? Check yes or no. (TEACHER NOTE: Students should check the correct box.) How do you know? (TEACHER NOTE: Students should write their explanations in the box.) Can the part below be used as a pattern in the path? Check yes or no. (TEACHER NOTE: Students should check the correct box.) How do you know? (TEACHER NOTE: Students should check the correct box.) How do you know? (TEACHER NOTE: Students should check the correct box.) How do you know?
- 3. The square shapes in the tricycle path can be grouped into sections as shown below. (TEACHER NOTE: Have students look at the sections shown.) The table on the next page shows the number of sections and the number of each type of square shape in those sections. Complete the table with the correct numbers. (TEACHER NOTE: Students should fill in all the blanks in the table with the correct numbers.)
- 4. Look at your table to answer these questions. (TEACHER NOTE: Have students look at their tables.) If you know the number of sections, how do you find the number of design shapes? (TEACHER NOTE: Students should write their explanations in the box.) If you know the number of sections, how do you find the number of solid shapes? (TEACHER NOTE: Students should write their explanations in the box.) If you know the number of solid shapes, how do you find the number of sections? (TEACHER NOTE: Students should write their explanations in the box.)
- 5. Look at your table again. (TEACHER NOTE: Have students look at their tables.) Circle the rule that can be used to find the number of solid shapes. (TEACHER NOTE: Students should circle the correct rule.) Explain why the rule you chose works. (TEACHER NOTE: Students should write their explanations in the box.)

6. (TEACHER NOTE: This task may be a stretch for some students. If you feel it is beyond the scope of your class, you may omit it, use it as enrichment, or use it as a whole class or small group activity.)

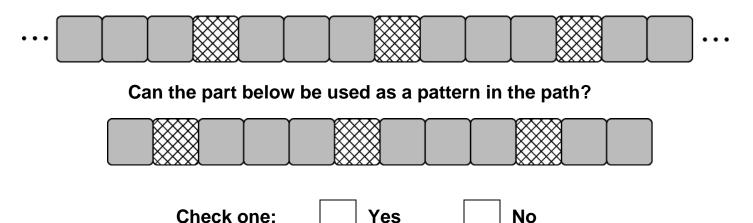
Look at part of the table below. (TEACHER NOTE: Have students look at their tables.) If there are *n* sections, how many solid shapes are there? (TEACHER NOTE: Students should write a correct expression using *n* on the line.) If there are *n* sections, how many design shapes are there? (TEACHER NOTE: Students should write a correct expression using *n* on the line.)

Student Response Sheet Everyday Patterns in Algebra



Name	:	Date:			
	Ms. Randolph's class is helping the local preschool build a path for children to ride their tricycles. The path has two types of square shapes that make a pattern.				
	Solid	Design			
	Part of the path is shown below				
	 Circle the set of square sh blanks to continue the pat 				
	a.	b.			
	c .	d.			
	Describe the pattern of the	e path.			

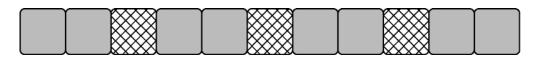
2. Another part of the path is shown below.



How do you know?



Can the part below be used as a pattern in the path?



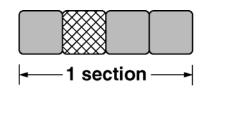
Check one:

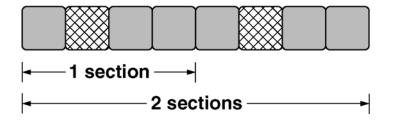
Yes

No

How do you know?

3. The square shapes in the tricycle path can be grouped into sections as shown below.





The table on the next page shows the number of sections and the number of each type of square shape in those sections. Complete the table with the correct numbers.

Number of Sections	Number of	Number of
1	3	1
2	6	2
3	9	3
4		
5		
6		
:	•	:
11		
:		:
		25
:	•	:
	90	
	93	

4. Look at your table to answer these questions:

?

a. If you know the number of sections, how do you find the number of ?

- b. If you know the number of sections, how do you find the
 - number of

c. If you know the number of ______, how do you find the

number of sections?

5. Look at your table again. Circle the rule that can be used to find the number of Number of = Number of + 2 a. = Number of Number of - 2 b. Number of = Number of × 3 C. = Number of Number of ÷ 3 d.

Explain why the rule you chose works.



6. Look at part of the table below.

Number of Sections	Number of	Number of
1	3	1
2	6	2
3	9	3
:		:

n	

a. If there are *n* sections, how many

are there?



Learning and Teaching Considerations

Task 1:

- A) Be sure that students understand the concept of a repeating pattern and how a pattern is extended or continued.
- **B**) Be sure that students understand that the core of the repeating pattern is the shortest string of elements that repeats. The teacher could use manipulatives such as colored tiles to create a pattern, show two iterations of the pattern, and then ask students to use the pattern blocks to show how to continue the pattern and explain their thinking.
- C) Students may have the misconception that the core of the repeating pattern is shorter or longer than the shortest string of elements that repeats. If this happens, ask them to convince you and the class that their pattern will always work
- **D**) Students may have the misconception that the pattern does not continue after one or two iterations. Be sure students understand that patterns are continuous and go on infinitely.

Task 2:

- A) Be sure that students understand the core of the repeating pattern is the shortest string of elements that repeats. The teacher could use manipulatives such as colored tiles to create a pattern, show two iterations of the pattern, and then ask students to use the pattern blocks to show how to continue the pattern and explain their thinking.
- **B**) Students may show evidence of attempting to fit a pattern or to draw a pattern over the path picture.
- C) Students may have the misconception that the core of the repeating pattern is shorter or longer than the shortest string of elements that repeats.
- D) Students may show evidence of counting solids between design shapes.

Task 3:

- A) Be sure students understand that the number patterns found in number sequences are based on a particular rule.
- **B**) Be sure students understand that a rule is something that will always work. For example, the student might notice a recursive pattern in which the previous number is operated on to get the next number.

- C) Be sure that students understand the relationship between the visual pattern and numeric component (the number of objects in each section). One column of the table or chart is the number of sections and the other columns are for recording how many objects are in each section. When looking for relationships, some students will focus on the table and others will focus on the physical pattern. It is important for students to see that whatever relationships they discover exist in both forms. For example, if the student finds the relationship in the table, challenge the students to see how that plays out in the visual pattern.
- **E**) Be sure that students understand that the number pattern appears in that order (in each column of the chart) and the numbers cannot be reversed or switched.
- F) Students may show evidence of multiplication or division by 3.
- G) Students may show evidence of repeated addition or subtraction of 3.

Task 4:

- A) Students may add 3 to the previous number to get the next number. This is an example of a "recursive" rule; that is, a rule that uses the previous term to get the next term.
- **B**) Students may multiply the section number by 3 to get each number in the middle column (which represents the solid shapes). This is an example of a "closed" rule; that is, a rule in which the term can be found directly from the row number.
- **C)** Students may multiply the previous number by 3 to get the next number. This is a mistake that mixes together a recursive rule and a closed rule.
- D) Students may answer in words, symbols (such as the addition or multiplication symbol), numbers, or by using manipulatives. They may also use number lines or charts. Be sure they understand that they can get the correct answer using any of these strategies. The teacher can also encourage them to link these strategies and/or representations to each other as a way to provide a convincing solution.
- E) If a student says or writes, "I just know," prompt him or her by saying something like "I'm glad you know, but it's important in math to be able to explain your answers so other people can understand what you're thinking."
- **F)** If a student says or writes, "I don't know," say something positive like "Let's start with what you do know about this problem." Students often know more than they think or say, and encouraging them to vocalize or write about that knowledge is all they need.

G) Students understand that the number of design shapes is equal to the number of sections and the number of solid shapes is 3 times the number of sections and the number of solid shapes divided by 3 equals the number of sections and number of design shapes.

Task 5:

- A) Students understand and are able to explain the correct rule.
- **B**) Students may provide some examples by multiplying some section numbers by 3.

Task 6:

- A) Students are able to write a general expression for the nth section (3*n* or 3 times n for part (a) and 1*n* or *n* for part (b)).
- B) If students say or write, "I don't know," ask them questions about the way they found the answers for tasks 3, 4, and 5 and encourage them to write the expression in words. For example, "What operation are you using?" "What number are you always multiplying by?" "What are we always multiplying 3 by?" "What information would *n* represent?"

Name: ANSWER KET

Date:

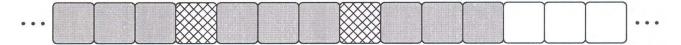
Ms. Randolph's class is helping the local preschool build a path for children to ride their tricycles. The path has two types of square shapes that make a pattern.



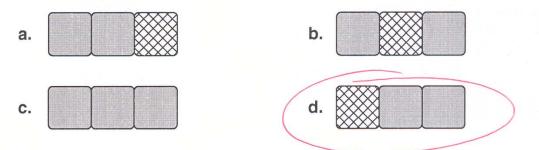
Design



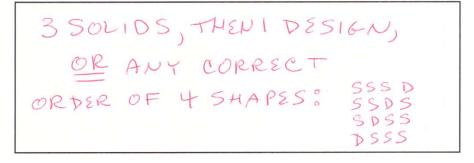
Part of the path is shown below.



1. Circle the set of square shapes that should go in the blanks to continue the pattern.



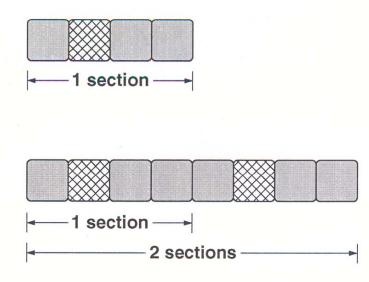
Describe the pattern of the path.



2. Another part of the path is shown below.

•••	
	Can the part below be used as a pattern in the path?
	Check one: Yes No How do you know?
	THERE ARE 3 SOLIDS TOGETHER FOLLOWED BY I DESIGN.
	ANSWERS MAY VARY
	Can the part below be used as a pattern in the path?
	Check one: Yes No How do you know?
ъ.	THERE ARE NEVER 3 SOLIDS TOGETHER.
	ANSWERS MAY VARY.

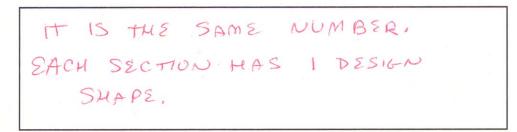
3. The square shapes in the tricycle path can be grouped into sections as shown below.



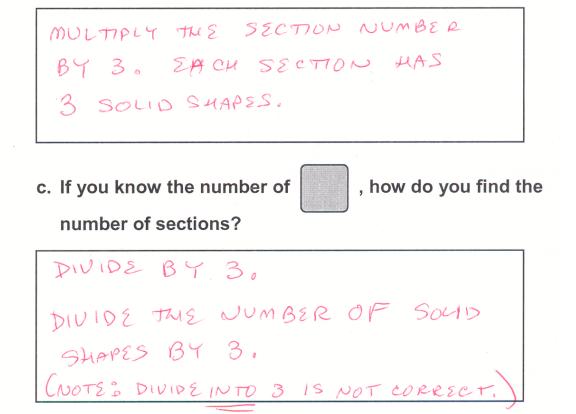
The table on the next page shows the number of sections and the number of each type of square shape in those sections. Complete the table with the correct numbers.

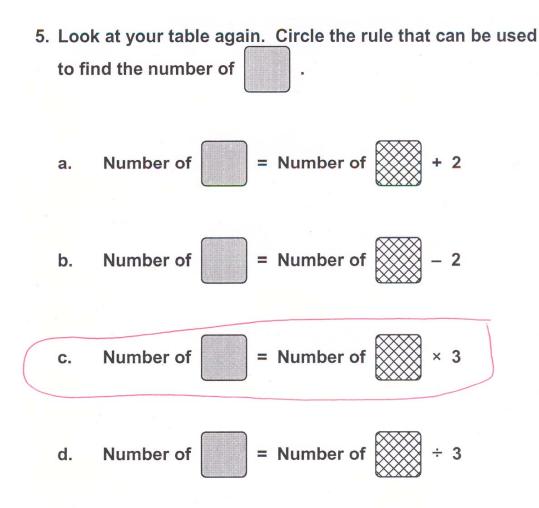
Number of Sections	Number of	Number of
1	3	1
2	6	2
3	9	3
4	12	4
5	15	5
6	18	6
•		:
11	33	()
:	:	:
25	75	25
•	•	•
30	90	30
31	93	31

- 4. Look at your table to answer these questions:
 - a. If you know the number of sections, how do you find the number of ?



b. If you know the number of sections, how do you find the number of ?





Explain why the rule you chose works.

EACH SECTION HAS 3 TIMES
AS MANY SOLIDS AS DESIGNS,
$$OR$$

 $3 = 1 \times 3$
 $6 = 2 \times 3$
 $9 = 3 \times 3$
 $R = 4 \times 3$ ETC.

6. Look at part of the table below.

Number of Sections	Number of	Number of
1	3	1
2	6	2
3	9	3
n	3 n	n

a. If there are *n* sections, how many

are there?



b. If there are *n* sections, how many



are there?

n

CATEGORY	4	3	2	1
Mathematical concepts	Response shows complete understanding of the mathematical concepts used to solve the problem(s).	Response shows substantial understanding of the mathematical concepts used to solve the problem(s).	Response shows some understanding of the mathematical concepts needed to solve the problem(s).	Response shows very limited understanding of the underlying concepts needed to solve the problem(s), OR the response is not written.
	Response shows evidence in ALL of the following tasks. <u>Task 1</u> . Student describes a correct path pattern containing 3 solids and 1 design. <u>Task 2</u> . Student is able to distinguish between the pattern that works and the pattern that doesn't work. <u>Task 3</u> . Student completes the last two rows of the table correctly. <u>Task 4</u> . Student shows evidence of understanding that the number of design shapes is equal to the number of solid shapes is 3 times the number of sections. <u>Task 5</u> . Student answers 3 <i>n</i> to part (a) and <i>n</i> to part (b).	Response shows evidence in only 4 of the tasks described.	Response shows evidence in only 3 of the tasks described.	Response shows evidence in 2 or fewer of the tasks described.

CATEGORY	4	3	2	1
Strategy and procedures	Student typically uses an efficient and effective strategy to solve the problem(s).	Student typically uses an effective strategy to solve the problem(s).	Student sometimes uses an effective strategy to solve the problem(s), but not consistently.	Student rarely uses an effective strategy to solve the problem(s).
	Response shows evidence in all of the following tasks. <u>Task 1</u> . Student may show evidence of markings in the path picture that begin to differentiate a pattern between solid and design. <u>Task 2</u> . Student may show evidence of attempting to fit pattern or to draw pattern over the path picture. Student may show evidence of counting solids between design shapes. <u>Task 3</u> . Student may show evidence of multiplication or division by 3. Student may show evidence of subtraction of 3. <u>Task 5</u> . Student may show evidence of filling numbers into the rules to see if they work.	Response shows evidence in only 3 of the tasks described.	Response shows evidence in only 1 or 2 of the tasks described.	Response shows no evidence of describing or showing a correct strategy or rule.

CATEGORY	4	3	2	1
Explanation and communication	Explanation is detailed and clear; uses appropriate terminology and/or notation.	Explanation is clear; uses some appropriate terminology and/or notation.	Explanation is a little difficult to understand, but includes critical components; shows little use of appropriate terminology and/or notation.	Explanation is difficult to understand, is missing several components, and does not use or include appropriate terminology and/or notation.
	Response shows evidence in ALL of the following tasks. <u>Task 1</u> . Student explains that each section of the path contains 3 solids and 1 design. The order may be different, depending on where the path begins. <u>Task 2</u> . Student explains that the first part can be used in the path because it could be put onto the right end and the pattern of 3 solids and a design continues. Student explains that the second part cannot be used in the path because there are sections that contain only 2 solids. <u>Task 4</u> . Student explains that each section contains 1 design shape, so the number of design shapes will equal the number of sections. Student explains that each section contains 3 solid shapes, so the number of solid shapes will equal 3 times the number of sections. Student explains that the number of sections will equal the number of solid shapes divided by 3. Student may say that part (c) is the opposite or reverse ("inverse" is the correct mathematical term) of part (b). <u>Task 5</u> . Student explains that the number of solids is equal to the number of design shapes times 3. Student may show multiplication equations.	Response shows evidence in only 3 explanations.	Response shows evidence in only 1 or 2 explanations.	Response shows no explanation.

CATEGORY	4	3	2	1
Mathematical accuracy	All or almost all of the steps and solutions have no mathematical errors.	Most of the steps and solutions have no mathematical errors.	Some of the steps and solutions have no mathematical errors.	Few of the steps and solutions have no mathematical errors.
	Student provides correct answers for ALL of the following tasks. <u>Task 1</u> . Student answers d. <u>Task 2</u> . Student answers yes for first part and no for second part. <u>Task 3</u> . Student completes the table, as shown on answer key. <u>Task 5</u> . Student answers c. <u>Task 6</u> . Student completes the table and answers 3 <i>n</i> for part (a) and <i>n</i> for part (b). (Note: 3 <i>n</i> is the preferred way to write the expression. However, 3 times <i>n</i> , $3 \times n$, and 3(n) are also acceptable. $3 + n$ is not acceptable.)	Student provides correct answers for only 4 of the tasks described in category 4.	Student provides correct answers for only 2 or 3 or the tasks described in category 4.	Student provides a correct answer for only 1 task or none of the tasks described in category 4

Scoring Notes Checklist

Task	Check Yes	Category
Task 1		
Describe a correct path pattern containing 3 solids and 1 design.		Concept
Shows evidence of markings in the path picture that begin to		Strategy
differentiate a pattern between solid and design. Explains that each section of the path contains 3 solids and 1 design. The order may be different depending, on where the path begins.		Explanation
Answers d.		Accuracy
Task 2		
Distinguishes between the pattern that works and the pattern that doesn't work.		Concept
Shows evidence of attempting to fit pattern or to draw pattern over the path picture, or shows evidence of counting solids between design shapes.		Strategy
Explains that the first part can be used in the path because it could be put onto the right end and the pattern of 3 solids and a design continues. Explains that the second part cannot be used in the path because there are sections that contain only 2 solids.		Explanation
Answers yes for first part and no for second part.		Accuracy
Task 3		
Completes the last two rows of the table correctly.		Concept
Shows evidence of multiplication or division by 3, or shows evidence of repeated addition or subtraction of 3.		Strategy
Completes the table, as shown on answer key.		Accuracy
Task 4		
Shows evidence of understanding that the number of design shapes is equal to the number of sections and the number of solid shapes is 3 times the number of sections.		Concept
Explains that each section contains 1 design shape, so the number of design shapes will equal the number of sections. Explains that each section contains 3 solid shapes, so the number of solid shapes will equal 3 times the number of sections. Explains that the number of sections will equal the number of solid shapes divided by 3. May say that part (c) is the opposite or reverse ("inverse" is the correct mathematical term) of part (b).		Explanation
Task 5		
Explain the correct rule.		Concept
Shows evidence of filling numbers into the rules to see if they work.		Strategy
Explains that the number of solids is equal to the number of design shapes times 3. May show multiplication equations.		Explanation
Answers c.		Accuracy

Task 6	
Answers 3 <i>n</i> to part (a) and <i>n</i> to part (b).	Concept
Completes table and answers $3n$ for part (a) and <i>n</i> for part (b). (Note: $3n$ is the preferred way to write the expression. However, 3 times <i>n</i> , $3 \times n$, and $3(n)$ are also acceptable.)	Accuracy

Analyzing Student Responses Protocol

The purpose of the Mathematics Challenges is to provide opportunities for students to develop and demonstrate understanding of important mathematical concepts and standards. They include extended responses, open-ended tasks, and tasks that require higher-order thinking skills. Because these types of tasks may be novel for students and they will have varying levels of understanding, the student responses will vary.

The guiding questions below were designed to assist you in analyzing your class' response to the Challenge and determining appropriate next steps for your teaching and learning. Responses to these questions are for your reflection and documentation and will not be collected.

Guiding Questions for Analyzing Student Responses to the Mathematics Challenges

1. When completing the Challenge, what did your students do well? How do you know?

2. When completing the Challenge, what did your students struggle with? How do you know?

3. When your students completed the Challenge, did they implement multiple correct solutions strategies? What insightful approaches to problem solving did you observe?

4. What, if any, patterns (e.g., common errors/misconceptions) did you observe across your student responses?

5. What questions or concerns did your students have when working through this Challenge or a particular task? Are these things you should address for the class as a whole?

6. What, if any, feedback did you provide to your class? How did you provide it?

7. What did you learn about your students' mathematical understanding based on their responses to this Challenge?

Reminders:

- After you have completed the Challenge with your class and responded to these Guiding Questions for Analyzing Student Responses, please complete the Challenge Feedback Log. A link to this Log is e-mailed to you each month. Responses will be used to improve the Challenges and to provide recommendations for teachers implementing the Challenges in future years.
- 2) Please provide copies of all student work to the Assessment Coordinator.