SITES-M Mathematics Challenge



Level: Grade Three

Standard: Geometry and Measurement

Learning Target: Focus on Shapes

Grade Level Expectations

GLE 0306.4.2 Understand and apply the concepts of congruence and symmetry.

Checks for Understanding

0306.4.4 Identify, create, and describe figures with line symmetry.

State Performance Indicators

SPI 0306.4.2 Determine if two figures are congruent based on size and shape.

SITES-M Mathematics Challenge Grade 3–Focus on Shapes Designing the Flag

The purpose of the Mathematics Challenges is to provide opportunities for students to develop and demonstrate understanding of important mathematical concepts and standards. Each Challenge includes a set of tasks that require higher-order thinking skills. Because these types of tasks may be new for students and they will have varying levels of understanding, the student responses will vary. The Challenges and guiding questions were designed to help teachers plan their implementation and elicit, analyze, and act on evidence of student understanding.

You will be able to choose which Mathematics Challenge Packet to implement each month, according to the learning needs of your students and your teaching context. Each packet contains all the materials necessary to implement the Mathematics Challenge including a grade-appropriate Challenge, the Mathematics Challenge Meeting Protocol, and the Guiding Questions for Analyzing Student Responses to Mathematics Challenges.

For each Challenge, you will complete a six step process of planning, implementation, and analysis and reflection.

Stage	Step	Task	
	Step 1.	Review the Mathematics Challenge Meeting Protocol	
Planning	Step 2.	Review and solve the Mathematics Challenge prior to your Professional Learning Community (PLC) meeting. Think about your responses to the guiding questions on the Meeting Protocol	
	Step 3.	Hold your PLC meeting and discuss your responses to the Guiding Questions on the Meeting Protocol	
Implementation	Step 4.	Implement the Mathematics Challenge with your class	
	Step 5.	For your own planning and documentation, respond to the Guiding Questions on the Analyzing Student Responses Protocol	
Analysis and Reflection	Step 6.	To help us improve the Challenges and to provide recommendations for teachers implementing them in future years, complete the Mathematics Challenge Feedback Log and provide copies of all student work to the Assessment Coordinator	

The Mathematics Challenge Process

SITES-M Mathematics Challenge Grade 3–Focus on Shapes Designing the Flag

Mathematics Challenge Meeting Protocol

Each month, your Professional Learning Community will meet to discuss the implementation of one Mathematics Challenge. In preparation for your monthly meeting, please print and review this month's Mathematics Challenge, solve all tasks within the Challenge, and think about the guiding questions below. These questions will be used to facilitate a group discussion regarding the implementation of the upcoming Mathematics Challenge.

Guiding Questions for Implementing the Mathematics Challenges

- 1. What is the title of the Challenge that you will use this month?
- 2. What skills or standards is this Challenge measuring?
- 3. Where does this Challenge fit within your curriculum? Within which unit?
- 4. At what point during the unit will you administer this Challenge (e.g., At the beginning of a unit to determine what students do or do not know, at the end of a unit to assess what students have or have not learned, in the middle of a unit to determine where to go next instructionally)?
- 5. How will your students complete this Challenge (e.g., individually, one-on-one, in small groups, as a class)? Why?
- 6. Are there any prerequisite skills, common misunderstandings, or vocabulary needs that you will have to address? What are they?
- 7. What difficulties do you anticipate your students will have with the Challenge? How will you address them?
- 8. Are these skills and difficulties different for special needs students, ELL students, etc.? How? Will you do anything different for these students? What?
- 9. How will you evaluate student responses (e.g., grade responses with the provided rubric, scan responses to identify common mistakes/misconceptions, have students evaluate one another's responses, have students evaluate their own response)?
- 10. What will student responses to this Challenge tell you about student understanding?
- 11. How might you use this evidence of student understanding to adapt your teaching and learning?
- 12. What other materials, resources, or support might you need? Where can you get them?
- 13. How can your colleagues assist you in the analysis of student understanding?
- 14. What other questions or concerns do you have about this Mathematics Challenge?

After you have implemented the challenge with your class, be sure to respond to the Guiding Questions on the Analyzing Student Responses Protocol.



Standard: Geometry and Measurement

Learning Target: Focus on Shapes

Claims:

Students should understand and be able to explain or demonstrate how to:

- ✓ Understand and apply the concepts of congruence and symmetry;
- ✓ Determine if two figures are congruent based on size and shape;
- ✓ Identify, create, and describe figures with line symmetry.

Task Preparation:

Each student will need a copy of the Student Response Sheet, a pencil, crayons, and possibly rulers (for drawing straight lines).

Stimulus Cards (Drawing or Word Description):

None

Manipulatives/Supplies:

Pencils, crayons, and possibly rulers (for drawing straight lines)

Cues/Directions:

Distribute student response sheets. Students should be directed to look at each figure carefully. Allow students time to answer.

Instruct students to follow along as you read aloud and say: The students of Shape Town are designing a new flag for City Hall. They want the flag to show congruence and symmetry.

- 1. Say: The grid below is full of four-sided polygons called quadrilaterals. (TEACHER NOTE: Have students look at the quadrilaterals on the grid. You may want to give students the opportunity to make observations about the quadrilaterals.) Shade all the quadrilaterals that are congruent to one another. (TEACHER NOTE: Students should color in the congruent quadrilaterals.) How can you use the grid to tell which quadrilaterals are congruent and which are not? (TEACHER NOTE: Students should write their explanations in the box.) Some quadrilaterals have line symmetry and some do not. Circle the quadrilaterals in the grid that do <u>not</u> have a line of symmetry (TEACHER NOTE: Students should circle the quadrilaterals that do not have a line of symmetry.) How did you decide which shapes to circle? (TEACHER NOTE: Students should write their explanations in the box.)
- 2. (TEACHER NOTE: You may want to hand out rulers to help students draw straight lines in this task and the next.) Jacob is using a quadrilateral in the shape of an arrow for a design on the flag. The arrow shape is on the grid below. The point of the arrow is marked A. (TEACHER NOTE: Have students look at the quadrilateral on the grid.) Three more points marked A are also on the grid. Draw three arrow shapes on the grid that are congruent to Jacob's design. Put the point of each arrow at an A. (TEACHER NOTE: Students should use their rulers to draw 3 arrow shapes on the grid with the point of each arrow being at an A.) How can you tell your arrows are congruent? (TEACHER NOTE: Students should write their explanations in the box.)
- 3. Luisa wants to use a shape that is symmetric to a diagonal line on the flag. Three polygons are drawn above the diagonal line on the grid below. Draw the other half of each polygon so that the diagonal line is the line of symmetry. (TEACHER NOTE: Students should use their rulers to draw the other half of each shape below the line of symmetry. Students may fold the paper along the diagonal line in order to help them with this task.) How did you decide where to draw your shapes? (TEACHER NOTE: Students should write their explanations in the box.)

- 4. The students finally decided on a design for the flag. The design is shown below. (TEACHER NOTE: Have students look at the design. You may want to give students the opportunity to make observations about the design.) The students say that the diagonal line that goes from one corner to the opposite corner is a line of symmetry for the flag design. Are they right? Check yes or no. (TEACHER NOTE: Students should check the correct box.) How do you know? (TEACHER NOTE: Students should write their explanations in the box.)
- 5. **(TEACHER NOTE:** This task may be a stretch for some students. If you feel it is beyond the scope of your class, you may omit it, use it as enrichment, or use it as a whole-class or small-group activity.**)**

Here is a picture of the flag of the state of Tennessee. (TEACHER NOTE: Have students look at the picture of the flag. You may want to give students the opportunity to make observations about the flag.) Write all the shapes you see in the picture of the flag (TEACHER NOTE: Students should write one shape on each line provided.) Which shapes are congruent? (TEACHER NOTE: Students should write the name of the congruent shapes on the line provided.) How can you tell? (TEACHER NOTE: Students should write their explanations in the box.) Here is a picture of the center of the flag. Draw a line of symmetry on the design. (TEACHER NOTE: Students should draw one of the three lines of symmetry on the design.)



Student Response Sheet

Designing the Flag

Name:		

Date: _____

The students of Shape Town are designing a new flag for City Hall. They want the flag to show congruence and symmetry.

1. The grid below is full of four-sided polygons called quadrilaterals. Shade all the quadrilaterals that are congruent to one another.



- a. How can you use the grid to tell which quadrilaterals are congruent and which are not?
- b. Some quadrilaterals have line symmetry and some do not. Circle the quadrilaterals in the grid that do <u>not</u> have a line of symmetry.

How did you decide which shapes to circle?

2. Jacob is using a quadrilateral in the shape of an arrow for a design on the flag. The arrow shape is on the grid below. The point of the arrow is marked A.

Three more points marked A are also on the grid. Draw three arrow shapes on the grid that are congruent to Jacob's design. Put the point of each arrow at an A.



How can you tell your arrows are congruent?

3. Luisa wants to use a shape that is symmetric to a diagonal line on the flag.

Three polygons are drawn above the diagonal line on the grid below. Draw the other half of each polygon so that the diagonal line is the line of symmetry.



How did you decide where to draw your shapes?



4. The students finally decided on a design for the flag. The design is shown below.



a. The students say that the diagonal line that goes from one corner to the opposite corner is a line of symmetry for the flag design. Are they right?

Check one:	Yes	No
How do you	know?	

5. Here is a picture of the flag of the state of Tennessee.



a. Write all the shapes you see in the picture of the flag.

b. Which shapes are congruent?

How can you tell?

c. Here is a picture of the center of the flag. Draw a line of symmetry on the design.



Learning and Teaching Considerations

Task 1:

- A) Be sure that students are able to identify five congruent shapes and describe characteristics of congruency (same shape and same size).
- **B**) If a student says or writes, "I just know," prompt him or her by saying something like "I'm glad you know, but it's important in math to be able to explain your answers so other people can understand what you're thinking."
- C) If a student says or writes, "I don't know," say something positive like "Let's start with what you do know about this problem." Students often know more than they think or say, and encouraging them to vocalize or write about that knowledge is all they need.
- **D**) Students may have the misconception that similar shapes that do not have the same side length or same angles are congruent.
- **E**) Be sure that students understand that if a shape can be folded on a line so that the two halves match, then it is said to have line symmetry.
- **F**) Students may have the misconception that the parallelogram has line symmetry because the opposite sides are equal and opposite angles are equal.
- **G**) Students may use grid markings and/or lengths of sides to describe congruence or line symmetry.
- **H**) Be sure that students are able to identify nonsymmetrical shapes and describe characteristics.

Task 2:

- A) Be sure that students are able to construct three symmetrical designs and describe how they know the designs are congruent.
- **B**) Students may recognize that they can translate and rotate the original design and/or coordinates of the original design.
- **C)** Students may realize that the inner point of the V-shape is two grid-marks below A. In this case, they may be able to count two marks to the right, to the left, and above the other three respective points A.
- **D**) Students may be able to describe how the congruent shapes were formed in relation to each point A.

Task 3:

- A) Be sure that students are able to construct three symmetrical designs and describe how it was done.
- **B**) Students may use grid markings in the description of symmetry.
- **C)** Students may be able to describe how the symmetrical shapes were formed in relation to each corner point of the given shapes.
- **D**) Students may fold along the diagonal line to help them draw the other half of each polygon so that the diagonal line is the line of symmetry.
- E) Teachers can encourage students who may need help to fold along the diagonal line.

Task 4:

- A) Students may recognize that when the flag is folded along the diagonal line, the designs will line up on top of each other.
- **B**) Students may have the misconception that the arrows are not symmetrical because they appear to be rotated.
- C) Teachers can encourage students who may need help to fold along the diagonal line.
- **D**) Students may count or measure on the grid to figure out whether the diagonal line that goes from one corner to the opposite corner is a line of symmetry for the flag design.

Task 5:

- A) Be sure that students are able to recognize the symmetry in the center of the flag by correctly drawing a line of symmetry.
- **B**) Be sure that students recognize that the stars in the flag are the same size and shape.
- **C)** Teachers can encourage students who may need help to fold along different diagonal lines, using trial and error.

Name: ANSWER KET

Date:

The students of Shape Town are designing a new flag for City Hall. They want the flag to show congruence and symmetry.

1. The grid below is full of four-sided polygons called quadrilaterals. Shade all the quadrilaterals that are congruent to one another.



a. How you can use the grid to tell which quadrilaterals are congruent and which are not?

The congruent figures are the same Size + the same shape. They are shaped like a kite with 2 short sides next to each other + 2 long sides next to each other. And they each had a grid marks inside. The other shapes did not have all of those, NOTE: ANSWERS MAY VARY, BUT A GOOD RESPONSE WILL MENTION GRIDMARKS,

b. Some quadrilaterals have line symmetry and some do not. Circle the quadrilaterals in the grid that do <u>not</u> have a line of symmetry.

How did you decide which shapes to circle?

These shapes can not be folded in half so that the sides match These shapes do not have a reflection line where one side of the line is a mirror of the other side

2. Jacob is using a quadrilateral in the shape of an arrow for a design on the flag. The arrow shape is on the grid below. The point of the arrow is marked A.

Three more points marked A are also on the grid. Draw three arrow shapes on the grid that are congruent to Jacob's design. Put the point of each arrow at an A.



How can you tell your arrows are congruent?

Answers will vary. A low level response will just say that they are the same size and the same shape, or that they look the same. A better response should discuss using the grid marks in some manner and positioning the 4 points of the figure on the grid. For example, a student might say that they counted down 2 marks from the A for one point. For the other 2 points, they may have counted over 2 both ways from the A and then down 4. Then they connected the points to form the shape. They could also talk of turning the paper to position the A at the top before determining the direction of the counts. 3. Luisa wants to use a shape that is symmetric to a diagonal line on the flag.

Three polygons are drawn above the diagonal line on the grid below. Draw the other half of each polygon so that the diagonal line is the line of symmetry.



How did you decide where to draw your shapes?



4. The students finally decided on a design for the flag. The design is shown below.



a. The students say that the diagonal line that goes from one corner to the opposite corner is a line of symmetry for the flag design. Are they right?



How do you know?



5. Here is a picture of the flag of the state of Tennessee.



a. Write all the shapes you see in the picture of the flag.

<u>RECTANGLE</u> <u>CIRCLE</u> <u>STAR (OR DODECAGON)</u> b. Which shapes are congruent? <u>3 STARS</u>

How can you tell?



c. Here is a picture of the center of the flag. Draw a line of symmetry on the design.



CATEGORY	4	3	2	1
Mathematical concepts	Response shows complete understanding of the mathematical concepts used to solve the problem(s).	Response shows substantial understanding of the mathematical concepts used to solve the problem(s).	Response shows some understanding of the mathematical concepts needed to solve the problem(s).	Response shows very limited understanding of the underlying concepts needed to solve the problem(s), OR the response is not written.
	Response shows evidence in ALL of the following tasks. <u>Task 1</u> . Student is able to identify 5 congruent shapes and to describe characteristics of congruency. Is able to identify 4 nonsymmetric shapes and to describe characteristics. <u>Task 2</u> . Student is able to construct 3 congruent shapes and describe how it was done. <u>Task 3</u> . Student is able to construct 3 symmetrical designs and describe how it was done. <u>Task 4</u> . Student recognizes and knows why the flag is symmetrical to the diagonal line. <u>Task 5</u> . Student recognizes the symmetry in the center of the flag by correctly drawing a line of symmetry.	Response shows evidence in only 4 of the tasks described; may exhibit the following errors. <u>Task 1</u> . Student may identify only some of the 5 congruent shapes; may identify only some of the 4 nonsymmetric shapes. <u>Task 2</u> . Student is able to construct only 1 or 2 congruent shapes. <u>Task 3</u> . Student is able to construct only 1 or 2 symmetrical shapes.	Response shows evidence in only 3 of the tasks described; may exhibit errors as described in category 3.	Response shows evidence in only 2 or fewer of the tasks described; may exhibit errors as described in category 3.

CATEGORY	Λ	2	2	1
Strategy and procedures	Student typically uses an efficient and effective strategy to solve the problem(s).	Student typically uses an effective strategy to solve the problem(s).	Student sometimes uses an effective strategy to solve problem(s), but not consistently.	Student rarely uses an effective strategy to solve problem(s).
	Response shows evidence in ALL of the following tasks. <u>Task 1</u> . Response shows evidence of pencil markings on grid to indicate counting or measuring between grid marks, or describes a strategy in the dialog boxes for determining both congruence and symmetry. <u>Task 2</u> . Response shows evidence of pencil markings on grid to indicate counting or measuring between grid marks, or describes a strategy in the dialog box for determining congruent shapes. <u>Task 3</u> . Response shows evidence of pencil markings on grid to indicate counting or measuring between grid marks, or describes a strategy in the dialog box for determining symmetrical shapes. <u>Task 4</u> . Response shows evidence of pencil markings on grid to indicate counting or measuring between grid marks, or describes a strategy in the dialog box for determining symmetrical shapes. <u>Task 4</u> . Response shows evidence of folding the flag along the diagonal line or evidence of pencil markings on grid to indicate counting, or describes a strategy in the dialog box for determining symmetry.	Response shows evidence in only 3 of the tasks described in category 4.	Response shows evidence in only 2 of the tasks described in category 4.	Response shows evidence in only 1 of the tasks described in category 4 or the response shows no evidence of strategy.

	_	-	-	-
CATEGORY	4	3	2	1
Explanation and communication	Explanation is detailed and clear; uses appropriate terminology and/or notation.	Explanation is clear; uses some appropriate terminology and/or notation.	Explanation is a little difficult to understand, but includes critical components; little use of appropriate terminology and/or notation.	Explanation is difficult to understand, is missing several components, does not use or include appropriate terminology and/or notation.
	Response shows evidence in ALL of the following tasks. <u>Task 1</u> . Student gives a detailed description that includes grid markings and lengths of sides about shape in part (a); describes decision about figures that do have symmetry in part (b). <u>Task 2</u> . Student uses the grid markings in the description of congruency; describes how the congruent shapes were formed in relation to each point A. <u>Task 3</u> . Student uses the grid markings in the description of symmetry; describes how the symmetric shapes were formed in relation to each corner point of the given shapes. <u>Task 4</u> . Student explains that when the flag is folded along the diagonal line, the designs will line up on top of each other. Student may also discuss grid marks. <u>Task 5</u> . Student explains that the stars in the flag are the same size and shape.	Response shows evidence in ALL of the tasks, but may lack detail in explanation, as evidenced by the following. <u>Task 1</u> . Student does not discuss grid marks or may say only that the shapes look the same. <u>Task 2</u> . Student does not discuss grid marks or relation of corner points to A. <u>Task 3</u> . Student does not discuss grid marks or relation of corner points to line of symmetry <u>Task 4</u> . Student does not discuss folding along the diagonal or grid marks.	Response shows evidence in only 2 or 3 explanations. Generally, student does not discuss grid marks.	Response shows evidence in 1 explanation from category 4, or response gives no explanations.

CATEGORY	4	3	2	1
Mathematical	All or almost all of the steps and	Most of the steps and	Some of the steps and	Few of the steps and
accuracy	solutions have no mathematical	solutions have no	solutions have no	solutions have no
	errors.	mathematical errors.	mathematical errors.	mathematical errors.
	Student provides correct answers for ALL	Student provides correct	Student provides correct	Student provides a correct
	of the following tasks.	answers for only 4 of the	answers for only 2 or 3 of	answer for only 1 task or
	Task 1. Student shades only the 5	tasks described in	the tasks described in	none of the tasks described
	congruent shapes, as shown on the	category 4.	category 4.	in category 4.
	answer sheet. Student circles only the 4			
	shapes that have a line of symmetry, as			
	shown on the answer sheet.			
	Task 2. Student draws 3 congruent			
	shapes, as shown on the answer sheet.			
	<u>Task 3</u> . Student completes the polygons			
	to obtain the correct symmetry, as shown			
	on the answer sheet.			
	Task 4 . Student answers yes and			
	nothing else.			
	Task 5 . Student answers rectangle,			
	circle, star. Student answers that the			
	stars are congruent. Student draws at			
	least one correct line of symmetry in part			
	(C).			

Scoring notes checklist

Task	Check Yes	Category
Task 1		
Identifies 5 congruent shapes and to describe characteristics of congruency; identifies 4 nonsymmetric shapes and describes characteristics.		Concept
Explains that the houses all have 4 sides and 4 corners. The third similarity can vary, but mentioning that all are quadrilaterals is a high-level response.		Strategy
Gives a detailed description that includes grid markings and lengths of sides about shapes in part (a), and describes decision about figures that do have symmetry in part (b).		Explanation
Shades only the 5 congruent shapes, as shown on answer sheet. Circles only the 4 shapes that have a line of symmetry, as shown on the answer sheet.		Accuracy
Task 2		
Constructs 3 congruent shapes and describes how it was done.		Concept
Evidence of pencil markings on grid to indicate counting or measuring between grid marks, or describes a strategy in the dialog box for determining congruent shapes.		Strategy
Uses the grid markings in the description of congruency, and describes how the congruent shapes were formed in relation to each point A.		Explanation
Draws 3 congruent shapes as shown on the answer sheet.		Accuracy
Task 3		
Constructs 3 symmetrical designs and describes how it was done.		Concept
Evidence of pencil markings on grid to indicate counting or measuring between grid marks, or describes a strategy in the dialog box for determining symmetric shapes.		Strategy
Uses the grid markings in the description of symmetry, and describes how the symmetrical shapes were formed in relation to each corner point of the given shapes.		Explanation
Completes the polygons to obtain the correct symmetry, as shown on the answer sheet.		Accuracy
Task 4		
Recognizes and knows why the flag is symmetrical to the diagonal line.		Concept
Evidence of folding the flag along the diagonal line or evidence of pencil markings on grid to indicate counting or measuring, or describes a strategy in the dialog box for determining symmetry.		Strategy
Explains that when the flag is folded along the diagonal line, the designs will line up on top of each other. May also discuss grid marks.		Explanation
Answers yes and nothing else.		Accuracy
Task 5		
Recognizes the symmetry in the center of the flag by correctly drawing a line of symmetry.		Concept
Explains that the stars in the flag are the same size and shape.		Explanation
Answers rectangle, circle, star. Answers that the stars are congruent. Draws at least one correct line of symmetry in part (c).		Accuracy

Analyzing Student Responses Protocol

The purpose of the Mathematics Challenges is to provide opportunities for students to develop and demonstrate understanding of important mathematical concepts and standards. They include extended responses, open-ended tasks, and tasks that require higher-order thinking skills. Because these types of tasks may be novel for students and they will have varying levels of understanding, the student responses will vary.

The guiding questions below were designed to assist you in analyzing your class' response to the Challenge and determining appropriate next steps for your teaching and learning. Responses to these questions are for your reflection and documentation and will not be collected.

Guiding Questions for Analyzing Student Responses to the Mathematics Challenges

1. When completing the Challenge, what did your students do well? How do you know?

2. When completing the Challenge, what did your students struggle with? How do you know?

3. When your students completed the Challenge, did they implement multiple correct solutions strategies? What insightful approaches to problem solving did you observe?

4. What, if any, patterns (e.g., common errors/misconceptions) did you observe across your student responses?

5. What questions or concerns did your students have when working through this Challenge or a particular task? Are these things you should address for the class as a whole?

6. What, if any, feedback did you provide to your class? How did you provide it?

7. What did you learn about your students' mathematical understanding based on their responses to this Challenge?

Reminders:

- After you have completed the Challenge with your class and responded to these Guiding Questions for Analyzing Student Responses, please complete the Challenge Feedback Log. A link to this Log is e-mailed to you each month. Responses will be used to improve the Challenges and to provide recommendations for teachers implementing the Challenges in future years.
- 2) Please provide copies of all student work to the Assessment Coordinator.