Common Core Mathematics Challenge

Next to Nothing?

Level: Grade Five

Domain: Numbers and Operations in Base Ten

Cluster: Understand the place value system.

Standards

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Read, write, and compare decimals to thousandths.
The purpose of the Mathematics Challenges is to provide opportunities for students to develop and demonstrate understanding of important mathematical concepts and standards. Each Challenge includes a set of tasks that require higher-order thinking skills. Because these types of tasks may be new for students and they will have varying levels of understanding, the student responses will vary. The Challenges and guiding questions were designed to help teachers plan their implementation and elicit, analyze, and act on evidence of student understanding.

Each packet contains all the materials necessary to implement the Mathematics Challenge including a grade-appropriate Challenge, the Mathematics Challenge Meeting Protocol, and the Guiding Questions for Analyzing Student Responses to Mathematics Challenges.

For each Challenge, you will complete a six step process of planning, implementation, and analysis and reflection.

### The Mathematics Challenge Process

<table>
<thead>
<tr>
<th>Stage</th>
<th>Step</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planning</td>
<td>Step 1</td>
<td>Review the Mathematics Challenge Meeting Protocol</td>
</tr>
<tr>
<td></td>
<td>Step 2</td>
<td>Review and solve the Mathematics Challenge prior to your Professional Learning Community meeting. Think about your responses to the guiding questions on the Meeting Protocol</td>
</tr>
<tr>
<td></td>
<td>Step 3</td>
<td>Hold your Professional Learning Community meeting and discuss your responses to the Guiding Questions on the Meeting Protocol</td>
</tr>
<tr>
<td>Implementation</td>
<td>Step 4</td>
<td>Implement the Mathematics Challenge with your class</td>
</tr>
<tr>
<td>Analysis and Reflection</td>
<td>Step 5</td>
<td>For your own planning and documentation, respond to the Guiding Questions on the Analyzing Student Responses Protocol</td>
</tr>
<tr>
<td></td>
<td>Step 6</td>
<td>To help us improve the Challenges and to provide recommendations for teachers implementing them in future years, complete the Mathematics Challenge Feedback Log and provide copies of all student work to the Assessment Coordinator</td>
</tr>
</tbody>
</table>
Common Core Mathematics Challenge
Number and Operations in Base Ten
Grade 5–Next to Nothing?

Mathematics Challenge Meeting Protocol

Your Professional Learning Community will meet to discuss the implementation of one Mathematics Challenge. In preparation for your meeting, please print and review the Mathematics Challenge, solve all tasks within the Challenge, and think about the guiding questions below. These questions will be used to facilitate a group discussion regarding the implementation of the upcoming Mathematics Challenge.

Guiding Questions for Implementing the Mathematics Challenges

1. What is the title of the Challenge that you will use?
2. What skills or standards is this Challenge measuring?
3. Where does this Challenge fit within your curriculum? Within which unit?
4. At what point during the unit will you administer this Challenge (e.g., At the beginning of a unit to determine what students do or do not know, at the end of a unit to assess what students have or have not learned, in the middle of a unit to determine where to go next instructionally)?
5. How will your students complete this Challenge (e.g., individually, one-on-one, in small groups, as a class)? Why?
6. Are there any prerequisite skills, common misunderstandings, or vocabulary needs that you will have to address? What are they?
7. What difficulties do you anticipate your students will have with the Challenge? How will you address them?
8. Are these skills and difficulties different for special needs students, ELL students, etc.? How? Will you do anything different for these students? What?
9. How will you evaluate student responses (e.g., grade responses with the provided rubric, scan responses to identify common mistakes/misconceptions, have students evaluate one another’s responses, have students evaluate their own response)?
10. What will student responses to this Challenge tell you about student understanding?
11. How might you use this evidence of student understanding to adapt your teaching and learning?
12. What other materials, resources, or support might you need? Where can you get them?
13. How can your colleagues assist you in the analysis of student understanding?
14. What other questions or concerns do you have about this Mathematics Challenge?

After you have implemented the challenge with your class, be sure to respond to the Guiding Questions on the Analyzing Student Responses Protocol.

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Domain: Number and Operations in Base Ten

Cluster: Understand the place value system

Standards:
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Read, write, and compare decimals to thousandths.

Task Preparation:
Each student will need a copy of the student response sheets.

Stimulus Cards (Drawing or Word Description):
None

Manipulatives/Supplies:
Pencils
Cues/Directions:
Distribute student response sheets. Students should be directed to look at each figure carefully. Allow students time to answer.

1. Instruct students to follow along as you read aloud and say: Each large square below has been divided into 100 smaller squares of equal size. Each large square represents 1 unit. Shade in 0.48 of the large square below. (TEACHER NOTE: Have students correctly shade in 0.48 of the square.) Shade in 0.7 of the large square below. (TEACHER NOTE: Have students correctly shade in 0.7 of the square.) Which number, 0.48 or 0.7, is bigger? (TEACHER NOTE: Have students check the correct box.) How do you know? (TEACHER NOTE: Students should write their explanation in the box.) Shade in the squares below to represent the sum below. (TEACHER NOTE: Have students correctly shade in 0.7 + 0.48 of the square.) What is the sum? (TEACHER NOTE: Students should write their correct answer on the line.)

2. Complete the table below by checking which number the decimal is closest to. The first one is done for you. (TEACHER NOTE: Have students correctly fill in the table.) Which decimal number in the table is closest to 0? (TEACHER NOTE: Students should write their correct answer on the line.) How do you know? (TEACHER NOTE: Students should write their explanation in the box.)

3. Four regions on the number line below are labeled. (TEACHER NOTE: Give students time to examine the number line.) Complete the table below by finding the region of the number line that has that number. The first one is done for you. (TEACHER NOTE: Have students correctly fill in the table.) Which number in the table is closest to 1? (TEACHER NOTE: Students should write their correct answer on the line.) How do you know? (TEACHER NOTE: Students should write their explanation in the box.)

4. Use the four numbers in the box above to write an addition problem with decimals so that the answer is as close to 0 as possible. (TEACHER NOTE: Students should write their answer in the box, this might be a stretch for some students.) How do you know that your answer is as close to 0 as possible? (TEACHER NOTE: Students should write their explanation in the box.)
5. Use the four numbers in the box above to write an addition problem with decimals so that the answer is as close to 1 as possible, but less than 1. (TEACHER NOTE: Students should write their answer in the box, this might be a stretch for some students.) Use the same numbers to write an addition problem with decimals so that the answer is as close to 1 as possible, but more than 1. (TEACHER NOTE: Students should write their answer in the box, this might be a stretch for some students.) Which of your answers is closer to 1? (TEACHER NOTE: Students should write their correct answer on the line.) How do you know? (TEACHER NOTE: Students should write their explanation in the box.)
1. Each large square below has been divided into 100 smaller squares of equal size. Each large square represents 1 unit.

   a. Shade in 0.48 of the large square below.

   b. Shade in 0.7 of the large square below.
c. Which number, 0.48 or 0.7, is bigger?

Check one:  

0.48  
0.7

How do you know?


d. Shade in the squares below to represent the sum below.

0.48 + 0.7

What is the sum?  ____________________________
2. Complete the table below by checking which number the decimal is closest to. The first one is done for you.

<table>
<thead>
<tr>
<th>Decimal Number</th>
<th>The decimal is closest to which number?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1.16</td>
<td></td>
</tr>
<tr>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>0.033</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>

a. Which decimal number in the table is closest to 0?

________________________

How do you know?
3. Four regions on the number line below are labeled.

![Number Line Diagram]

Complete the table below by finding the region of the number line that has that number. The first one is done for you.

<table>
<thead>
<tr>
<th>Decimal Number</th>
<th>The decimal is in which region?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.62</td>
<td>C</td>
</tr>
<tr>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>0.706</td>
<td></td>
</tr>
</tbody>
</table>

a. Which number in the table is closest to 1? __________

How do you know?
4.  

a. Use the four numbers in the box above to write an addition problem with decimals so that the answer is as close to 0 as possible.

b. How do you know that your answer is as close to 0 as possible?
5. **4 7 1 6**

a. Use the four numbers in the box above to write an addition problem with decimals so that the answer is as close to 1 as possible, but **less than 1**.

b. Use the same numbers to write an addition problem with decimals so that the answer is as close to 1 as possible, but **more than 1**.
c. Which of your answers is closer to 1? ______

How do you know?
Task 1:
A) Be sure that students understand that the positions of digits determine the value of such digits within the number. The base ten place-value system extends infinitely in two directions. Between any adjacent place values, the ratio of the left place to the right place is 10 to 1.

B) If a student says or writes, “I just know,” prompt him or her by saying something like “I’m glad you know, but it’s important in math to be able to explain your answers so other people can understand what you’re thinking.”

C) If a student says or writes, “I don’t know,” say something positive like “Let’s start with what you do know about this problem.” Students often know more than they think or say, and encouraging them to vocalize or write about that knowledge is all they need.

D) The teacher could encourage students to use manipulatives such as base ten blocks to explore decimals, by using the big cube (that is created with 100 units) to represent 1, rather than 1,000. The manipulatives can be used by students as a tool to reason and make sense of our base ten system.

E) Be sure that students understand that there are patterns to the way that numbers in our number system are formed. For example, in base ten, each decade has a symbolic pattern reflective of the 0-to-9 sequence. There are ten possible symbols that can be used as a digit in each place.

F) Student explanations may refer to the area model, to the placement of the digits, or to fractions.

Task 2:
A) Be sure that students understand that decimal numbers are another way of writing fractions.

B) Be sure that students have opportunities to think of decimals in terms of familiar fraction equivalents and are able to make this connection in a conceptual manner.

C) Students may draw pictures or use manipulatives.

D) Students may convert fractions to decimals or decimals to fractions.

E) Students may have the misconception that 0.033 has more digits than some of the others and is therefore not closest to zero.
Task 3:
A) Be sure that students are able to apply and discuss the related concepts of fractions, place value, and decimals in activities.

B) Students may write the numbers on the number line and/or convert fractions to decimals or decimals to fractions.

C) Students may have the misconception that the letters represent points on the number line rather than regions.

D) Students may have the misconception that 0.9 has less digits than some of the others and is therefore not the closest to 1.

E) Students may have the misconception that the number with more digits is the largest.

Task 4:
A) Students may explain that the smallest numbers need to go into the biggest place values.

B) Students may use trial and error by combining the 4 digits to achieve different sums.

C) Students may use the least numbers (2 and 3) in the biggest place value, the tenths, and the greatest numbers (9 and 5) in the smallest place value, the hundredths, to achieve the least sum.

Task 5:
A) Students may provide an explanation that discusses distance from 1, either by subtraction or by a comparison on the number line.

B) Students may use trial and error by combining the 4 digits to achieve different sums.

C) Students may subtract each sum from 1 or subtract 1 from each sum.
1. Each large square below has been divided into 100 smaller squares of equal size. Each large square represents 1 unit.

a. Shade in 0.48 of the large square below.

b. Shade in 0.7 of the large square below.
c. Which number, 0.48 or 0.7, is bigger?

Check one:  

- [ ] 0.48
- [x] 0.7

How do you know?

MORE OF THE SQUARE IN (b) IS SHADOED.

OR \(0.7 > 0.48\)

OR \(\frac{70}{100} > \frac{48}{100}\)

d. Shade in the squares below to represent the sum below.

\[0.48 + 0.7\]

What is the sum? \[1.18\]
2. Complete the table below by checking which number the decimal is closest to. The first one is done for you.

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</tr>
<tr>
<td>1.16</td>
<td></td>
</tr>
<tr>
<td>0.22</td>
<td>✓</td>
</tr>
<tr>
<td>0.05</td>
<td>✓</td>
</tr>
<tr>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>1.02</td>
<td></td>
</tr>
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<td></td>
</tr>
<tr>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>0.033</td>
<td>✓</td>
</tr>
<tr>
<td>0.15</td>
<td>✓</td>
</tr>
</tbody>
</table>

a. Which decimal number in the table is closest to 0?

**0.033**

How do you know?

**IF YOU LOOK AT ALL THE NUMBERS AS HUNDREDTHS, THIS IS JUST A LITTLE MORE THAN 3. THE NEXT SMALLEST IS 5 HUNDREDTHS OR 0.05. 0.033 IS 33 HUNDREDTHS.**

**Note:** Student may also convert to thousandths.
3. Four regions on the number line below are labeled.

A
B
C
D

0
1
\frac{1}{4}
\frac{1}{2}
\frac{3}{4}
1

Complete the table below by finding the region of the number line that has that number. The first one is done for you.

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<tr>
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<td>0.004</td>
<td>A</td>
</tr>
<tr>
<td>0.1</td>
<td>A</td>
</tr>
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<td>D</td>
</tr>
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<td>0.9</td>
<td>D</td>
</tr>
<tr>
<td>0.49</td>
<td>D</td>
</tr>
<tr>
<td>0.3</td>
<td>B</td>
</tr>
<tr>
<td>0.51</td>
<td>C</td>
</tr>
<tr>
<td>0.706</td>
<td>C</td>
</tr>
</tbody>
</table>

a. Which number in the table is closest to 1? \[\boxed{.9}\]

How do you know?

IT HAS THE MOST TENTHS OF ALL THE NUMBERS IN THE TABLE.

\[.9 = \frac{9}{10} \text{ or } \frac{90}{100} \text{ or } \frac{900}{1000}\]
4.

\[
\begin{array}{c}
9 \\
2 \\
5 \\
3 \\
\end{array}
\]

a. Use the four numbers in the box above to write an addition problem with decimals so that the answer is as close to 0 as possible.

\[
\begin{array}{c}
.259 \\
+ .3 \\
\hline
.559
\end{array}
\]

or

\[
\begin{array}{c}
.359 \\
+ .2 \\
\hline
.559
\end{array}
\]

\textbf{NOTE:} \quad .25 \\
\begin{array}{c}
+ .39 \\
\hline
.64
\end{array}
\text{ is not as close to 0 as } .559

b. How do you know that your answer is as close to 0 as possible?

\textbf{YOU WANT THE SMALLEST DIGITS IN THE BIGGEST PLACE VALUE. SO THE 2 AND 3 SHOULD BE IN THE TENTHS PLACE.}
5.  

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

a. Use the four numbers in the box above to write an addition problem with decimals so that the answer is as close to 1 as possible, but less than 1.

\[
\begin{array}{cc}
.74 + .16 & \text{or} & .76 + .14 \\
\frac{.90}{.90}
\end{array}
\]

b. Use the same numbers to write an addition problem with decimals so that the answer is as close to 1 as possible, but more than 1.

\[
\begin{array}{cc}
.417 + .6 & \text{or} & .617 + .4 \\
\frac{1.017}{1.017}
\end{array}
\]

**Note:** \[
\frac{.41 + .67}{1.08} \text{ is not as close as } 1.017 \]
c. Which of your answers is closer to 1? 1.017

How do you know?

.9 IS 1 TENTH AWAY
FROM 1

1.017 IS LESS THAN 1
TENTH AWAY FROM 1.

SO 1.017 IS CLOSER.

\[
\begin{array}{c}
1.0 \\
- .9 \\
\hline
.1
\end{array}
\quad
\begin{array}{c}
1.017 \\
- .1 \\
\hline
.017
\end{array}
\quad
.1 > .017
\]

Note: Students can give a correct reason for their answers even if the answers are wrong.
### GOLD

A student in this category would
- demonstrate complete understanding of concepts addressed in the challenge;
- execute procedures completely and correctly in order to give relevant responses to the questions;
- provide good work that supports their answers which may contain few minor errors if any; and
- communicate effectively how the problem was solved when asked to provide such evidence.

A student in this category has provided good evidence that they are proficient at the mathematical concepts addressed within the challenge.

### GREEN

A student in this category would
- demonstrate nearly complete understanding of concepts addressed in the challenge;
- execute most procedures completely and correctly in order to give relevant responses to the questions with some possible minor errors;
- provide good work that supports their answers which may contain few minor errors; and
- communicate how the problem was solved when asked to provide such evidence, but the explanation may not be detailed or complete.

A student in this category has provided good evidence that they are proficient at the mathematical concepts addressed within the challenge, but may have some problems with their explanation of their understanding of them.
<table>
<thead>
<tr>
<th>Yellow</th>
<th>A student in this category would</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• demonstrate limited understanding of concepts addressed in the challenge;</td>
</tr>
<tr>
<td></td>
<td>• execute some procedures completely and correctly in order to give relevant responses to the questions, but with some errors;</td>
</tr>
<tr>
<td></td>
<td>• provide acceptable work that supports their answers, but may be incomplete or contain some major errors; and</td>
</tr>
<tr>
<td></td>
<td>• communicate ineffectively about how the problem was solved when asked to provide such evidence with the explanation being incomplete, lacking detail, or missing.</td>
</tr>
<tr>
<td></td>
<td>A student in this category has provided some evidence that they are familiar with the mathematical concepts addressed within the challenge, but some of the skills need to be reviewed for the student to achieve proficiency.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Red</th>
<th>A student who falls into this category would</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• demonstrate insufficient understanding of concepts addressed in the challenge;</td>
</tr>
<tr>
<td></td>
<td>• be unable to execute most procedures completely or correctly in order to give relevant responses to the questions;</td>
</tr>
<tr>
<td></td>
<td>• provide incorrect or limited work to support their answers with major errors; and</td>
</tr>
<tr>
<td></td>
<td>• have missing or poor explanations when asked to provide them.</td>
</tr>
<tr>
<td></td>
<td>A student in this category has provided little to no evidence that they are proficient at the mathematical concepts addressed within the challenge. Remediation of skills is needed and should be planned for the student.</td>
</tr>
</tbody>
</table>
Common Core Mathematics Challenge
Number and Operations in Base Ten
Grade 5–Next to Nothing? Rubric

Analyzing Student Responses Protocol

The purpose of the Mathematics Challenges is to provide opportunities for students to develop and demonstrate understanding of important mathematical concepts and standards. They include extended responses, open-ended tasks, and tasks that require higher-order thinking skills. Because these types of tasks may be novel for students and they will have varying levels of understanding, the student responses will vary.

The guiding questions below were designed to assist you in analyzing your class’ response to the Challenge and determining appropriate next steps for your teaching and learning.

Guiding Questions for Analyzing Student Responses to the Mathematics Challenges

1. When completing the Challenge, what did your students do well? How do you know?

2. When completing the Challenge, what did your students struggle with? How do you know?

3. When your students completed the Challenge, did they implement multiple correct solutions strategies? What insightful approaches to problem solving did you observe?
4. What, if any, patterns (e.g., common errors/misconceptions) did you observe across your student responses?

5. What questions or concerns did your students have when working through this Challenge or a particular task? Are these things you should address for the class as a whole?

6. What, if any, feedback did you provide to your class? How did you provide it?

7. What did you learn about your students’ mathematical understanding based on their responses to this Challenge?